A Column Generation Heuristic for the General Vehicle Routing Problem

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Abstract. This paper presents a column generation heuristic for the general vehicle routing problem (GVRP), a combined load acceptance and rich vehicle routing problem incorporating various real-life complexities. Computational experiments show that proposed column generation heuristic is competitive with heuristics previously presented for the GVRP.

Key words: Vehicle Routing Problem, Pickup and Delivery Problem, Heterogeneous Vehicle Fleet, Column Generation

1 Introduction

The general vehicle routing problem (GVRP) is a rich transportation problem generalising the vehicle routing problem with time windows, the vehicle routing problem with heterogeneous vehicle fleet and multiple depots, and the pickup and delivery problem. Real-life applications of the GVRP arise, for example, in the road transport of air-cargo, see [10]. So-called *road feeder services* are charged to transport air-cargo between large freight hubs and smaller airports. Airlines typically accept additional load until shortly before departure of the aircraft. Due to different capacities between aircrafts and trucks a change in capacity utilisation of aircrafts may result in the need for additional trucks to forward air-cargo to the final destination. Thus, providers of road feeder services must be capable of simultaneously deciding whether to serve new transportation requests and determining new routes for the trucks in in order to guarantee that all confirmed transportation requests are served.

The GVRP has first been described by [11] who present variable neighbourhood search and large neighbourhood search algorithms capable of tackling the various complexities of the GVRP. This paper presents a column generation heuristic for the GVRP and compares its competitiveness over computation time with the best approach presented by [11]. The column generation approach presented in this paper is particularly characterised by its simplicity. Computational experiments show that, despite its simplicity, competitive results can be obtained, especially if computation time is limited.

The remainder of this paper is organised as follows. Section 2 surveys related work. Section 3 gives a verbal description of the GVRP and Section 4 presents the set partitioning formulation used for the column generation approach presented in Section 5. Computational experiments are reported in Section 6. Eventually, Section 7 gives some concluding remarks.

2 Related Work

The vehicle routing problem and many of its variants are extensively studied in [17]. Surveys on construction heuristics and metaheuristics for the vehicle routing problem with time windows are presented by [2] and [3]. Column generation has been successfully applied to the vehicle routing problem with time window by [8]. [5] presents a column generation approach for the the dynamic vehicle routing problem with time windows. This approach, like the approach presented in this paper, generates new columns by removing or inserting new transportation requests into existing columns. Vehicle routing problems with heterogeneous vehicle fleets are surveyed by [1]. Recently, [6] presented a column generation approach for the heterogeneous fleet vehicle routing problem.

Another important generalisation of the vehicle routing problem is the pickup and delivery problem surveyed by [12] and [7]. Column Generation has been successfully applied to various rich pickup and delivery problems. [15] presents a column generation approach for the general pickup and delivery problem introduced by [14]. [18] presents a column generation approach for a rich pickup and delivery problem including a set of practical complications such as a heterogeneous vehicle fleet, multiple pickup/delivery time windows and driver's working hour regulations as imposed by the US Department of Transportation.

As can be seen, column generation has been successfully applied to a variety of transportation problems including various different complexities. This indicates that the column generation approach is particularly suitable for the GVRP presented by [11].

One of the major differences between the GVRP and most other vehicle routing problems discussed in the literature is that in the GVRP not all transportation requests must be served by self-operated vehicles. Thus, the GVRP requires for combined load acceptance and vehicle routing decisions. Combined load acceptance and routing approaches for the travelling salesman problem have been surveyed by [9]. Only few attempts have been made to tackle extensions of this problem. Among them is the approach by [16] who study the pickup and delivery selection problem.

This paper seeks to demonstrate the competitiveness of column generation for the GVRP when only a limited amount of time is available for optimisation. Column generation is often used to determine near-optimal solutions without taking particular care on the time required to calculate these solutions. Integer solutions are often only computed when no further columns with negative reduced costs can be found. Therefore, solutions found throughout the column generation process may be infeasible and cannot be used as comparison with other approaches, in particular heuristics. Due to the potentially large time until no further columns with negative reduced costs can be found, column generation approaches have been viewed critically if a trade-off between computation time and solution quality is of high importance. For example, [13] reports that *"a significant weakness is their slow execution time"*. However, column generation has also been successfully applied to dynamic problems, e.g. by [15] and [5]. This paper shows that when integer solutions are calculated in every iteration, the column generation approach presented in this paper can determine high quality solutions for the GVRP especially when only a limited amount of computation time is available.

The next sections will give a verbal formulation of the GVRP and a set partitioning formulation used for the column generation approach presented in this paper. A network flow type formulation of the GVRP can be found in [11].

3 The General Vehicle Routing Problem

In the general vehicle routing problem (GVRP) a transportation request is specified by a nonempty set of pickup, delivery and/or service locations which have to be visited in a particular sequence by the same vehicle, the time windows in which these locations have to be visited, and the revenue gained when the transportation request is served. Furthermore, some characteristics can be specified which constrain the possibility of assigning transportation requests to certain vehicles due to compatibility constraints and capacity constraints. At each of the locations some shipment(s) with several describing attributes can be loaded or unloaded. In contrast to many other commonly known routing problems, not all transportation requests have to be assigned to a vehicle. Instead, a so-called *make-or-buy* decision is necessary to determine whether a transportation request should be assigned to a self-operated vehicle (make) or not (buy).

A fleet of heterogeneous vehicles is available to serve transportation requests. The vehicles can have different capacities, as well as different travel times and travel costs between locations. Instead of assuming that each vehicle becomes available at a central depot, each vehicle is given a start location where it becomes available at a specific time and with a specific load. Furthermore, vehicles do not necessarily have to return to a central depot and for each vehicle a final location is specified, which has to be reached within a specific time and with a specific load. Each vehicle may have to visit some locations in a particular sequence between leaving its starting and reaching its final location. All locations have to be visited within a specific time window. If the vehicle reaches one of these locations before the beginning of the time window, it has to wait.

A tour of a vehicle is a journey starting at the vehicles start location and ending at its final location, passing all other locations the vehicle has to visit in the correct sequence, and passing all locations belonging to each transportation request assigned to the vehicle in the correct respective sequence. A tour is *feasible* if and only if for all orders assigned to the tour compatibility constraints hold and at each point in the tour time window and capacity restrictions hold.

The objective is to find distinct feasible tours maximising the profit, which is determined by the accumulated revenue of all served transportation requests, reduced by the accumulated costs for operating these tours.

4 The Set Partitioning Problem

Let $\mathcal{V}$ denote the set of vehicles and $\mathcal{O}$ the set of transportation requests. Let $\mathcal{T}_v$ denote the set of feasible tours for vehicle $v \in \mathcal{V}$ and let $\mathcal{T} := \bigcup_{v\in\mathcal{V}} \mathcal{T}_v$. For each tour $\theta \in \mathcal{T}$ the cost of the tour is denoted by c_θ. The total revenue of a tour $\theta \in \mathcal{T}$ is denoted by p_θ and can be calculated by summing up the revenue p_o associated to each transportation request $o \in \mathcal{O}$ served by the tour. For all transportation requests $o \in \mathcal{O}$ and all tours $\theta \in \mathcal{T}$ let $\delta_{o\theta}$ be a binary constant indicating whether transportation request o is served by tour θ ($\delta_{o\theta} = 1$) or not ($\delta_{o\theta} = 0$). Furthermore, let x_θ denote a binary variable indicating whether tour θ is used in the solution ($x_\theta = 1$) or not ($x_\theta = 0$). The GVRP can be represented as a set partitioning problem (SPP)

$$\text{minimise} \sum_{\theta\in\mathcal{T}} (c_\theta - p_\theta) x_\theta$$

subject to

$$\sum_{\theta\in\mathcal{T}} \delta_{o\theta} x_\theta \leq 1 \text{ for all } o \in \mathcal{O},$$

$$\sum_{\theta\in\mathcal{T}_v} x_\theta = 1 \text{ for all } v \in \mathcal{V},$$

$$x_\theta \in \{0, 1\} \text{ for all } \theta \in \mathcal{T}.$$

In this formulation, the objective is to maximise the profit which is determined by the difference between total revenue gained and accumulated costs for operating the tours. For notational reasons the maximisation problem is formulated as an equivalent minimisation problem. The first equation ensures that each transportation request is served at most once and the second equation ensures that for each vehicle exactly one tour is used in the solution. The practical complexities are not explicitly shown in the SPP, instead, all these complications are embedded in the columns of the formulation, which correspond to the tours of the vehicles.

5 The Column Generation Heuristic

As the SPP generally has a vast number of columns it can not be solved directly. Instead, a restricted version of this problem which only contains subsets of the columns is solved and additional columns are generated when needed. Let $\mathcal{T}'_v \subseteq$

$\mathcal{T}_v$ denote the subset of columns for vehicle $v \in \mathcal{V}$ and let $\mathcal{T}' := \bigcup_{v\in\mathcal{V}} \mathcal{T}'_v$. The restricted set partitioning problem (RSPP) is

$$\text{minimise} \sum_{\theta\in\mathcal{T}'} (c_\theta - p_\theta) x_\theta$$

subject to

$$\sum_{\theta\in\mathcal{T}'} \delta_{o\theta} x_\theta \leq 1 \text{ for all } o \in \mathcal{O},$$

$$\sum_{\theta\in\mathcal{T}'_v} x_\theta = 1 \text{ for all } v \in \mathcal{V},$$

$$x_\theta \in \{0,1\} \text{ for all } \theta \in \mathcal{T}'.$$

Let $x = (x_\theta)_{\theta\in\mathcal{T}'}$ be a feasible solution of the linear relaxation of RSPP and let $u = (u_o)_{o\in\mathcal{O}}$ and $w = (w_v)_{v\in\mathcal{V}}$ be the associated dual values. The reduced costs of a tour θ are

$$\text{reduced costs } := (c_\theta - p_\theta) - \sum_{o\in\mathcal{O}} \delta_{o\theta} u_o - w_v$$

where v denotes the vehicle associated to tour θ. From linear programming duality it is known that x is optimal with respect to the linear relaxation of SPP if and only if for each tour $\theta \in \mathcal{T}$ the reduced costs are nonnegative. If for any vehicle $v \in \mathcal{V}$ a feasible tour can be found with negative reduced costs, we know that the current solution is not optimal with respect to the linear relaxation of SPP and we can add this tour to $\mathcal{T}'_v$. The linear relaxation of RSPP can then be solved again with the modified set of tours. In order to keep the number of columns in the RSPP small, tours with reduced costs of more than a threshold Δ can be removed from the RSPP.

The column generation heuristic can now be described as follows:

1. Find initial sets $\mathcal{T}'_v$ containing a feasible solution x.
2. Solve the linear relaxation of RSPP.
3. Remove tours with reduced costs of more than Δ.
4. Find feasible tours with negative reduced costs and add them to the sets $\mathcal{T}'_v$.
5. Find an integer solution to RSPP.
6. If no new tours were found stop, otherwise continue with step 2.

In step 1 the auction algorithm for the GVRP described in [11] is used to determine an initial feasible solution of the GVRP. For each vehicle $v \in \mathcal{V}$ we set $\mathcal{T}'_v := \{\theta_v\}$ where θ_v denotes the tour of v in the initial solution. For solving the linear relaxation of RSPP in step 2 the commercial solver CPLEX (Version 9.1) is used. The threshold used in step 3 is initially set to $\Delta := 1000$. In order to constrain the number of columns in the RSPP this threshold is reduced by 10% whenever the number of columns exceeds 20000.

Feasible tours with negative reduced costs are found in step 4 by removing or inserting a transportation request from or to a tour θ with reduced costs of zero. The reduced costs of such a tour θ' are

$$(c_{\theta'} - p_{\theta'}) - \sum_{o \in \mathcal{O}} \delta_{o\theta'} u_o - w_v$$

$$= (c_{\theta'} - p_{\theta'}) - \sum_{o \in \mathcal{O}} \delta_{o\theta'} u_o - w_v - \underbrace{\Big((c_\theta - p_\theta) - \sum_{o \in \mathcal{O}} \delta_{o\theta} u_o - w_v\Big)}_{=0}$$

$$= \begin{cases} (c_{\theta'} - c_\theta) + (p_o + u_o) & \text{if } o \text{ is removed from } \theta \\ (c_{\theta'} - c_\theta) - (p_o + u_o) & \text{if } o \text{ is inserted to } \theta. \end{cases}$$

As removals and insertions only require local observations, the search for tours with negative reduced costs can be performed very efficiently, see e.g. [4]. If no tour with negative reduced costs is found by removing or inserting a transportation request from/to a tour with reduced cost of zero, new tours are found by subsequently removing a transportation requests from a tour with reduced cost of zero and inserting another transportation request to the same tour. It must be noted that our computational experiments reveal that combined removals and insertions are only rarely required.

The algorithm continues with step 5 after at most 100 new tours with negative reduced cost are generated or all removals and insertions are evaluated. In step 5 the MIP-solver of CPLEX (Version 9.1) is used to find an integer solution of RSPP. This generally takes little time. It must be noted that the calculation of an integer solution could also be only done just before termination of the algorithm. However, to be able to abort the program at any time and to compare the best results obtained until that time this step is required.

6 Computational Experiments

In order to evaluate the column generation heuristic, test cases were generated as follows. A fleet of heterogeneous vehicles is generated where some of the vehicles have refrigerated cargo bodies and some are manned by two drivers. All vehicles are en-route when planning starts and each vehicle becomes available at some time of the first day of the planning horizon of one week. All vehicles eventually have to return to their final destination.

Full and half truckload shipments are randomly generated such that the frequency distribution illustrated in Figure 1 is achieved. Transportation requests are generated by randomly selecting one full or half truckload shipment or by combining two half truckload shipments with identical pickup or delivery location. Some of the transportation requests require a vehicle with refrigerated cargo body, some require a vehicle manned by two drivers. The length of the time windows at each pickup or delivery location is set to the same value τ, i.e. either 2 hours or 12 hours.

Fig. 1. Distribution of pickup and delivery locations

Travel distances and travel costs are proportional to the geographical distance. Vehicles with refrigerated cargo bodies and vehicles manned by two drivers are more expensive than vehicles with standard cargo bodies and those manned by one driver. The revenue of transportation requests is set to double the costs of the cheapest vehicle capable of transporting the shipments. That is, the shippers are not only willing to pay for the transport itself, but also for the return trip to the start location.

Figure 2 shows the results of computational experiments performed on a personal computer with AMD Athlon processor with 400 MHz. It compares the results of the column generation approach (CG) compared to the large neighbourhood search algorithm with related removals (LNS) presented by [11]. It can be seen that for small problems with 50 vehicles and 250 transportation requests, CG clearly outperforms LNS and produces superior solutions with respect to the computation time available. For larger problems with 100 or 200 vehicles CG appears to be slightly outperforming LNS. Most solution values after ten minutes of computation are better for CG, however, with increasing computation time LNS can reduce the gap and even produce better solutions.

7 Concluding Remarks

This paper presents a column generation heuristic for the GVRP which is particularly characterised by its simplicity. Despite this simplicity the column generation approach is competitive with heuristic methods previously presented for the GVRP. The column generation approach appears to be particularly effective if only limited computation time is available until which a solution must be found.

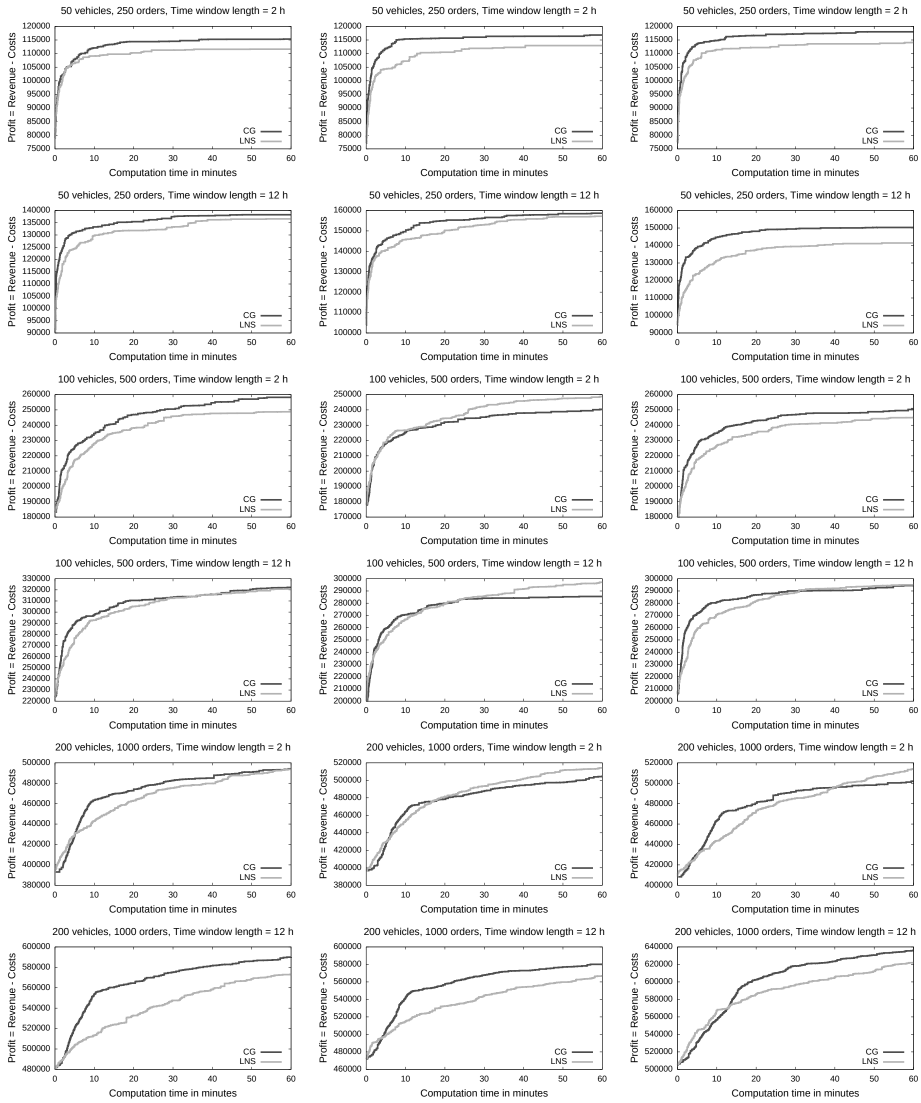

Fig. 2. Results

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